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THE STABILITY OF MOTION OF AN ELONGATED RIGID BODY OF REVOLUTION IN AN ELASTOPLASTIC MEDIUM WITH FLOW SEPARATION†

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The stability of rectilinear motion by inertia of slender rigid bodies of revolution in an elastoplastic medium is investigated qualitatively. The mathematical model of two-dimensional perturbed motion is based on the isolated-element method and hypotheses of flow separation, and reduces to an autonomous system of ordinary differential equations when there are unknown boundaries. Lyapunov's method is used to find bifurcation points of the stationary solution both with continuous flow and with cavitation with small asymmetrical zones of separation on the lateral surface of the body.

In experiments on the penetration of rigid bodies of revolution into strong media, distortion of the trajectory of motion (a rotation of up to 180°) has been observed. The issue on the stability of motion is important for the development of technologies based on the method of dynamic penetration [1, 2] but has been studied little only simulation calculations are known [3]. The present paper is based on solutions of model problems of the high-velocity elastoplastic flow around bodies [4–7] and on experiments [8]. Then, empirical parameters of well known models of local interaction are functionally related to the characteristics of the medium and the geometry of the body, which makes it possible to isolate additional small quantities and to reduce the number of parameters; in particular, friction does not enter into the equations for perturbations. The hypotheses of separation are based on observations of the motions of bodies in low-strength media and consist of the assumption that ideal separation occurs from the edges of the mid-section (it is observed at low velocities). For non-ideal separation, resulting from an increase in velocity and the presence of initial stresses, an empirical angle of separation is introduced.

The stability of rectilinear motion of a body is investigated depending on the portion of the centre of mass its cone the angle of separation and the velocity of motion, assuming that the perturbed motion is plane-parallel. The limits of the parameters which ensure stability in the case of continuous flow and the widening of these limits when asymmetrical small-area zones of separation appear near to the edges, corners or smooth points on the body surface are indicated. Configurations that are promising in the stability sense and absolutely unstable configurations are described. The model also makes it possible to assess the moments of deflection, the lateral forces, the safety margin, etc.

1. PHYSICAL DESCRIPTION OF THE PROBLEM

A rigid elongated body of revolution moves at high speed inertia in an unbounded isotropic and homogeneous elastoplastic medium. As the scale of length in the longitudinal and transverse directions we shall adopt the length of the body L and the maximum radius of its cross-section r_m, respectively. We shall put $R = R(l) = r/r_m$, the dimensionless equation of the generator of the lateral surface of the body in a cylindrical coordinate system R, ϕ, l rigidly connected to the body and with a local rectangular coordinate system $x = l_c - l, y = R\cos\phi, z = R\sin\phi$, where l is the distance from the tip of the body, and $l = l_c$ and $R = 0$ are the coordinates of the centre of mass. The conditions for the body to be thin

$$\varepsilon = r_m / L \ll 1, \quad \varepsilon\beta \ll 1, \quad \beta = R' = dR/dl, \quad 0 < l < 1 \tag{1.1}$$

are considered to be satisfied in almost all cases, with the possible exception of a small neighbourhood of the blunt nose part of the body $0 < l < l_1 \ll 1$, where we shall give a separate description.

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Remark. Bodies tapered at a small angle are not optimum from the viewpoint of strength under the action of transverse bending forces. On the other hand, even a sewing needle can be regarded as a blunt body, since its point (similar in shape to a paraboloid) is blunt with a characteristic radius $\sim 0.1r_m$.

At the initial instant of time $t = 0$, the velocity vector υ_0 of the centre of mass of the body and the angular velocity of rotation Ω_0 about this centre are specified. We shall assume that rotation begins in a plane formed by the velocity vector υ_0 and the body axis there is no yawing or other rotations). Then, the trajectories of motion of points of the body will be planar, if it also possessed axial symmetry with respect to dynamic properties.

We shall denote by $\mathbf{v} = (\upsilon_x, \upsilon_y, 0)$, $\Omega = (0, 0, \Omega)$, $\mathbf{V} = \mathbf{v} + \Omega \times (x, y, z)$ and $V_n = \mathbf{nV} = \varepsilon\delta\upsilon_x$ the vectors of the current translational, angular and total velocities and the velocity normal to the of the body surface where

$$\delta = \beta - a\cos\phi, \quad a = -\omega x - \eta, \quad \omega = \Omega L/(\varepsilon \upsilon_x), \quad \eta = \upsilon_y/(\varepsilon \upsilon_x)$$

The dimensionless functions ω and η—the angular velocity and angle of attack—are normalized so that they can take values $O(1)$. The mass m of the body will be expressed in terms of the dimensionless length of a cylinder of equivalent mass and mid-section and the average density of the body ρ_1 by means of the formula $m = \pi r_m^2 L l_e \rho_1$. An incompressible medium will be characterized by a density ρ_0, the shear modulus μ and the dynamic von Mises yield stress τ_d. For plastically compressible (porous) media it can be assumed that the medium was packed at a considerable distance from the body and that ρ_0 is its density in the packed state.

According to known results [5, 6], a plastic zone with a contour-like boundary and a "greater" radius $\sqrt{\mu/\tau_d}R(l)$ is formed near the contour of the body. Close to the contour, intense shear flow will be observed, with the flow separation from the edges of the rear part, and also, possibly, with separation on the lateral surface of the body.

A cavity retains its shape well in elastoplastic media, and, at low speeds, separation occurs practically under a condition of tangency: $V_n = 0$, i.e. $\delta = 0$. We shall term this condition the criterion of ideal separation. When the flow velocity increases and/or when there are considerable initial stresses in the medium σ_{ij}^0, this condition is not satisfied. Then, we shall adopt a generalized criterion, according to which the lines of separation on the body surface are determined from the condition that the angle $\delta = V_n/(\varepsilon\upsilon_x)$ should be equal to the critical quantity β_*:

$$\delta^* \equiv \delta - \beta_*(\sigma_{ij}^0, \upsilon_x) = 0 \tag{1.2}$$

($\delta^* > 0$ means contact, and $\delta^* < 0$ means separation).

With ideal separation $\beta_* = 0$. The separation angle β_* increases as the velocity increases and decreases as the initial compressive stresses increases (it may become negative). It must be determined from special experiments.

The piercing of a layer of a medium by a sphere with a polished surface can serve as such an experiment: the wetted part of the sphere surface will become mat, and, form the position of the boundary between the mat and polished surfaces, the angle of separation is determined (this idea is due to Yu. K. Bivin).

Similar empirical models of separation were used earlier in engineering calculations of the non-one-dimensional motion of a missile in the ground.

We shall confine ourselves to considering the conditions when there is no attachment of the jet on the surface of the body S. We shall designate the wetting zone by S_+ ($\delta^* > 0$) and the separation zone by; S_- ($\delta^* < 0$); $S = S_+ + S_-$. The vector of the contact stresses will be determined by the ordinary sum of the contributions of the hydrodynamic and strength terms [4–7]:

$$\sigma = \tau_s \mathbf{n}_\tau - \sigma_n \mathbf{n}, \quad \sigma_n = C_x \rho_0 V^2/2 + b_0 \tau_d \quad (S_+); \quad \sigma = 0 \quad (S_-)$$

where τ_s = const is the plastic friction ($\tau_s \leq \tau_d$) and $\mathbf{n}_\tau$ is the unit tangential vector in the direction of slip [in the approximation considered, $\mathbf{n}_\tau = (-1, 0, 1)$].

We shall divide the surface S into elements, each of which is approximated by a surface of canonical shape (a sphere, a cone or a cylinder), and specify the coefficients C_x and b_0, for example, from the solutions of model problems of steady flow [4–7] and from experiments [8]. The frontal surface $S_\perp$ under small perturbations of flow along the axis, will also remain entirely wetted, and on it, as on a unit element, $C_x = C_\perp$ and $b_0 = b_\perp$ = const.

When processing the dynamic penetration tests data [8] it was found that, for cones with semiaperture angles of 15–90°, the coefficient $C_\perp$ is similar to the calculated and experimental values in hydrodynamics. The quantity $b_\perp$ depends extremely weakly on the shape: at the angles of 15–90° it changes by 22% and differs roughly by one-third from the value of b_0 calculated from the formula for the maximum normal stress σ_n at the stagnation point of elastoplastic flow around a sphere [7]. Then, the value of $b_\perp$ on the surface $S_\perp$, and also the quantity $b_0 = b_f$ on the shallow part of the wetted lateral surface $S_f = S_+ - S_\perp$, where conditions (1.1) are satisfied and $\varepsilon\delta \ll 1$, can be approximated by the formulae [4–7]

$$b_\perp = \frac{4\sqrt{3}}{9}\left(4\ln\frac{\sqrt{3}\mu}{\tau_d}+1\right), \quad b_f = \ln\frac{4\mu}{\tau_d} - 1$$

For typical values of $\mu/\tau_d = 10^2 - 10^3$, we obtain $b_f = 5$–7 and $b_\perp = 16$–24, so that, in the asymptotic sense, we shall assume in what follows that $b_f^2, b_\perp^2 \gg 1$. According to results obtained in [4–6], on the surface S_f we shall have $C_x = C_f\varepsilon^2\delta^2$. The coefficient C_f, generally speaking, depends on the geometry of the body. Thus, the pressure is non-uniformly distributed along the generator of a thin cone of finite length, around which an ideal fluid flows. However, it is equalized in an elastoplastic medium, and there are grounds for putting $C_f =$ const and, according to the "tangential cone" method, we can take [6] $C_f = \ln(\mu/\tau_d) + 2.55$ provided that $\varepsilon\delta \ll (\tau_d/\mu)^{1/2}$ ($\varepsilon\delta < 10^{-2}$, i.e. for very small angles of inclination of a section of the surface to the flow), or for adopting a value consistent with experiments for clayey media—$C_f = 2.9$ ($\varepsilon\delta \sim 10^{-1}$).

The yield stress τ_d is a parameter of the process. Its measured values in dynamic experiments on the penetration of bodies into clayey media [8] are only 1.5–3 times higher than the corresponding static yield stress whereas behind the shock wave front (or in the elastic forerunner immediately before the front) they may differ from this value by an order of magnitude. The cause of the difference in τ_d and τ_s may be heating up of the medium about the contour from friction or preheating of the body (for example, to $T > 100$°C for ice).

The above assumptions and estimates are difficult to check directly, so the values of C_x, b_0, τ_d and τ_s must therefore be determined from control experiments. It is important to note that, in the adopted model of "angular" separation, the physical condition of the start of contact $\sigma_n = 0$ proves not to be satisfied. This is the cost of the approximate model of local interaction (either the "tangential cone" method, as in aerodynamics, or the isolated element method, as in mathematics). The contradiction disappears if σ_n is understood to mean the average value on a surface element of finite length; on the other hand, this indicates the need to improve the model.

We shall introduce additional dimensionless variables and parameters

$$\varkappa = \frac{c^2}{v_x^2}, \quad \xi: \ d\xi = \frac{v_x dt}{\zeta L}$$

$$c^2 = \frac{2b_f\tau_d}{\varepsilon^2\rho_0 C_f}, \quad \zeta = \frac{2l_e}{\rho C_f}, \quad \rho = \frac{\rho_0}{\rho_1}, \quad j = \frac{mL^2}{I}$$

where I is the principal moment of inertia of transverse rotation, and $\varkappa$ is the ratio of the strength and inertial drag forces, which varies during decelerated motion of the body in the range $\varkappa_0 < \varkappa < \infty$. When $\varkappa \ll 1$, inertia predominates. This is normally the region of supersonic speeds, where the mode employed in not always realistic. When $\varkappa \gg 1$ it is possible to ignore the effect of inertia. The order of magnitude of $\varkappa$ is determined not only by the speed but also by the strength of the medium, and it is possible to ensure a value of $\varkappa_0 \sim 1$ for low speeds. For a ground of average dynamic strength $\tau_d = 5 \times 10^6$ Pa penetration of a steel cone with an aperture angle of 15°, the condition $\varkappa \approx 1$ means that $V \approx 700$ m/s.

2. MATHEMATICAL STATEMENT OF THE PROBLEM

From the equations of motion of a body in terms of $\varkappa$, η, ω and ξ it follows that, with certain additional constraints on the nature of the blunting, the derivative $d\varkappa/d\xi = o(\varepsilon^2\varkappa)$, i.e. it is asymptotically small, whereas the derivatives of the perturbations η and ω are quantities of the order of $\varkappa$ with developed separation from the lateral surface. This has a clear physical meaning: the generalized lateral drag and the moment of the forces Stability of motion of an elongated rigid body of revolution in an elastoplastic medium predominate over the axial drag for slender bodies. In the case of continuous flow of a strong medium, as in the case of flow of a liquid around a body, the derivatives $\eta^{\bullet}$, $\omega^{\bullet} = O(1)$ are in the main also determined solely by the "hydrodynamic" component of the normal stress σ_n. In the following approximations, corrections will appear that take account of the influence of the strength, of the order of $\varkappa\varepsilon^2$, and they may make the main contribution as $\varkappa \to \infty$. Thus, the equations become invalid for describing continuous flow at the final stage of motion, but this will have only a slight effect on the main characteristics of motion (the penetration depth and the maximum values of the resulting forces). The determination of these corrections involves the solution in an accurate statement of model problems of the flow around slender bodies at an angle of attack and during their rotation.

Consequently, in a limited time interval ξ, the variation in $\varkappa$ can be ignored compared with the values of η and ω: the axial velocity "freezes". Physically, this approximation will be more accurate for "heavy" bodies ($\rho \ll 1$), low-strength media and relatively high speeds. Then, the determination of the perturbations η and ω reduces to solving the Cauchy problem for an autonomous system of two ordinary differential equations

$$d\eta / d\xi = f_\eta - \zeta\omega, \quad d\omega / d\xi = jf_\omega; \quad (\eta, \omega) = (\eta_0, \omega_0), \quad \xi = 0$$
$$(f_\eta, f_\omega) = \frac{1}{\pi}\int_{l_1}^{1} (1, l_c - l)\Phi R\, dl \tag{2.1}$$

In the derivation, terms $O(\varepsilon^2)$ are neglected. The resulting distributed lateral forces $\Phi(l)$ are determined by conditions of the inequality type:

1. $a > |\beta^*|$—the separation zone is localized about the meridian $\phi = 0$;

$$\Phi = 2a\beta(\pi - \phi^*) + \Psi, \quad \Psi = 2\{\varkappa + \beta^2 + a^2(2 + q^2)/3 - \beta\beta^*\}\sqrt{1 - q^2}$$
$$\phi^* = \arccos q, \quad q = \beta^*/a, \quad \beta^* = \beta - \beta_*$$

2. $a > |\beta^*|$—separation is localized about the meridian $\phi = \pi$; $\Phi = 2a\beta\phi^* - \Psi$;
3. $|a| \leqslant \beta^*$—continuous flow about the parallel: $\Phi = 2\pi\beta a$;
4. $|a| \leqslant -\beta^*$—complete separation along the parallel considered: $\Phi = 0$.

The right-hand sides of Eqs. (2.1) depend on the parameters $\varkappa$, ζ, l_c and j and the body shape $R(l)$. Despite linearization with respect to the parameter ε, the mathematical model possesses non-trivial non-linear properties. The bifurcation points of the trivial solution of these equations $\eta = \omega = 0$ will be determined below.

3. INVESTIGATION OF STABILITY LOCALLY

Equations (2.1) will be linear mainly in the vicinity of the singular point $\eta = \omega = 0$ if the body contour is non-singular: the function $\beta(l)$ is not identically equal to zero in any range of values of l in the segment [0, 1] and is piecewise-continuously differentiable. According to Lyapunov's method [9], we shall investigate the behaviour of solutions of the corresponding linearized system of equation.

$$dH/d\xi = AH, \quad H = (\eta, \omega), \quad A = A_{ij}$$
$$H = C_1 \exp(\lambda_1\xi) + C_2 \exp(\lambda_2\xi), \quad \lambda_{1,2} = B \pm \sqrt{B^2 - D} \tag{3.1}$$

where C_i are constant vectors and λ_i are the roots of the characteristic equation. The nature of the solution depends on the invariants of the matrix A

$$2B = A_{11} + A_{22}, \quad D = A_{11}A_{22} - A_{12}A_{21}$$

In most of the cases investigated below, $B < 0$. Then, only the sign of the determinant D is subject to analysis. For $0 < D < B^2$, both roots $\lambda_{1,2}$ are negative, and stability is ensured locally (a singular point a stable node). When $D > B^2$, the roots are complex and Re $\lambda_i < 0$ (a stable focus). When $D < 0$, one of the roots is positive (a saddle point)—motion is unstable in the Lyapunov sense. The equation $D = 0$ sefines the stability limits in parameter space.

4. THE STABILITY CRITERION IN THE CASE OF CONTINUOUS FLOW AROUND LATERAL SURFACE OF A BODY AND WITH FLOW SEPARATION FROM THE EDGE

Suppose the critical angle of separation β_* is negative, and the slender body has a strictly monotonically increasing radius of cross-section in the direction from the tip of the body to its rear part, and, furthermore, suppose the angle of inclination of the generator to the body axis of exceeds the critical angle of separation $0 \leqslant \beta_* < \beta(l)$. Then, for perturbations so small that $\beta_* \geqslant a$, there is a continuous flow around complete lateral surface (separation occurs from the rear edge), the strength of the medium has no influence on the lateral loads, and the integrals in (2.1) are taken in the explicit form

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$$f_\eta = -p_0\eta + p\omega, \quad f_\omega = p\eta + (2l_c p_1 - p_2 - l_c^2 p_0)\omega$$

$$p = p_1 - l_c p_0, \quad p_i = \int_{l_1}^{1} l^i dR^2, \quad i = 0, 1, 2$$

Equations (2.1) are identical to (3.1) and it can be proved that the following inequalities hold

$$p_0 > p_1 > p_2, \quad p_0 p_2 - p_1^2 > 0 \Rightarrow B < 0 \tag{4.1}$$

Following the algorithm set out in Section 3, we arrive at the following assertion.

Assertion 1. In the case of continuous flow around a body with an angle of inclination of the generator greater than the critical angle of separation β_* $(0 \leqslant \beta_* < \beta(l))$, its rectilinear motion will be Lyapunov stable if the distance l_c of its centre of mass from the tip of the body is less than the critical value l_g and is unstable if $l_c > l_g$. The quantity l_g is given by the formula

$$l_g = 1 - \left\{\zeta(p_0 - p_1) + p_1^2 - p_2 p_0\right\}/(\zeta p_0) \tag{4.2}$$

The dependence of l_g on the density ratio is unexpected. Note that this quantity is not the same as the distance of the centre of pressure from the tip of the body l_p, which is given by the formula (the integrals were evaluated for the case of a cone)

$$l_p = l_c - \frac{f_\omega}{f_\eta} = \int_{l_1}^{1} \beta\Phi R l dl \left(\int_{l_1}^{1} \beta\Phi R dl\right)^{-1} = \frac{2\eta - (\tfrac{3}{2} - 2l_c)\omega}{3\eta - (2 - 3l_c)\omega}$$

as adopted in engineering aerodynamics on the basis of analysis solely of the signs of the moments of the forces in the case of a steady flow. Furthermore, the considerable dependence of l_p on perturbations can be seen from this formula.

5. THE FLOW SEPARATION AT POINTS OF SMOOTHNESS OR AT A CORNER OF THE GENERATOR

Let the unperturbed motion of a body with a convex contour be characterized by the flow separation along the parallel $l = l_w$, $0 < l_w < 1$ at points of smoothness of the contour according to the condition $\beta^*(l_w) = 0$, so that the body surface at $l < l_w$ $(\beta^*(l) > 0)$ is wetted, while at $l > l_w$ $(\beta^*(l) < 0)$ it is stress-free. We shall examine small perturbations η and ω such that the maximum displacements of points of separation l_+ l_w and $l_- > l_w$ on the meridians $\phi = 0$ and π in different directions from the unperturbed position is also small. These points are related to the perturbations by the equality $\beta(l_\pm) - \beta_* = \pm a$, and, by virtue of the condition for the function $\beta(l)$ to be small and smooth, we have

$$\beta(l_+) - \beta(l_-) = 2[(l_w - l_c)\omega - \eta] \approx h(l_+ - l_-), \quad h = |\beta'(l_w)|$$

The asymptotically principal part of function Φ takes the form

$$\Phi = \begin{cases} 0, & l > l_- \\ 2\pi a\beta, & l < l_+ \\ (\varkappa + \beta_*^2)\sqrt{1 - z^2}, & l_+ < l < l_-; \quad z = (l - l_w)/(l_- - l_w) \end{cases}$$

On separation from a corner $l = l_w$, $0 < l_w \leqslant 1$, the condition $\beta_- \leqslant \beta_* \leqslant \beta_+$ is satisfied where β_- and β_+ are the angles of inclination of the contour to the left and right of the corner (the flow proceeds from right to left with respect to the body). If the inequality is strict, small perturbations do not lead to asymmetry of the flow, and the stability analysis in Section 4 holds, taking into account the renormalization of the length of the wetted section, When $\beta_* = \beta_{\mp}$, in the one-sided vicinity of the corner it is possible for asymmetrical separation zones to occur for almost any perturbations. Then, in the formulae given above, l_+ or l_- must be replaced by l_w. An additional contribution to the lateral force and moment will be obtained by integrating the expressions in (2.1) with respect to the region $l_+ < l < l_-$

$$\Delta f_{\eta,\omega} = \psi(u_w\omega - \eta)(1, -u_w), \quad \psi = \frac{\varkappa + \beta_*^2}{eh} R(l_w) > 0, \quad u_w = l_w - l_c$$

where $e = 1$ when the contour is smooth and, $e = 2$ in the case of a corner and, accordingly, the left or right-hand derivative $\beta'_\pm$ is taken when determining h. By selecting the initial perturbations, it is possible to ensure the length of the zone to be small over a sufficiently long time interval. Then, for a body with an angle of inclination of the generator such that $0 \leqslant \beta_* < \beta(l)$ with $0 < l < l_w$, Assertion 1 holds when the critical quantity l_g is replaced by

$$l_s = l_g + \frac{\psi(p_0 l_w - p_1)(\zeta + p_0 l_w - p_1)}{\zeta p_0(\psi + p_0)}, \quad p_i = 2\int_{l_1}^{l_w} l^i R\, dR \tag{5.1}$$

where l_g is defined in terms of the new values of p_i by formula (4.2). Inequalities (4.1) hold when renormalization is taken into account, and therefore $l_s > l_g$. If $\psi \to \infty$ and $l_w \to 1$, then the asymptotics

$$l_s \to 1 + \zeta^{-1}(p_0 + p_2 - 2p_1) > 1$$

holds and we conclude that, for ψ, ζ and l_w values such that

$$\psi > \frac{\zeta(p_0 - p_1) + p_1^2 - p_0 p_2}{\zeta(l_w - 1) + p_0 l_w^2 + p_2 - 2p_1 l_w} > 0$$

the motion is stable for any position of the centre of mass over the length of the body. The growth of ψ and an increase in the margin of stability are ensured by increasing the curvature of the contour at separation points and by reducing the axial velocity (stabilization with an increase in the effect of strength forces, $\varkappa \to 0$). Cavitation reduces the drag considerably, stabilizes the axial velocity and increases the path length are to the reduction in the areas of the mid-section and the wetted surface. However, for any fixed ψ we have $l_s \to 0$ if $l_w \to 0$. From this it follows that control of the stability by changing the parameter ψ becomes less effective as cavitation develops, and when there is complete separation from the flunted (but smooth) tip of the body and $l_c > l_1$, the state of the dynamical system proves to be unstable irrespective of the values of the other parameters.

It can be shown that stricter allowance for the asymmetry of the flow around the nose part of the body is then necessary. However, a comparative analysis reveals that the ratio of the moment of the axial force about the centre of mass M_a to the moment of the lateral forces M, caused by the asymmetry of the separation, is a quantity of the order εR_1, where it is assumed that the magnitude of the displacement of the point of application of the force along the y axis $d = d_\eta \eta$, where $d_\eta = O(1)$. Consequently, for elongated and smooth bodies, the asymmetry of the flow around the blunted nose part can be neglected compared with the effects of the flow around the lateral surface. When the flow separation occurs from a corner in the generator close to the nose part of the body, such an analysis becomes necessary.

For numerical estimates when $l_w = 1$ wee shall use the inequalities

$$0 < p_0 < 1, \quad 0 < p_1 < {}^2/_3, \quad 0 < p_2 < {}^1/_2, \quad {}^1/_3 < l_c < 1$$

where extreme values are achieved for a cone and for a cylinder. In the case of continuous flow, in the range of values of relative density $0 < \rho < 1$, the following ranges of variation in l_g correspond to them

$$0 < l_g < {}^2/_3 + \chi/18, \quad 0 < \chi < 4, \quad \chi = \zeta^{-1}; \quad {}^2/_3 < l_g < {}^8/_9 \quad \text{(cone)}$$

Taking into account the effect of separation in the rear part of the body contour, we obtain

$$\psi(1+\chi)/(1+\psi) < l_s < 1 + \chi/6 - (^1/_3 + \chi/9)/(1+\psi)$$

Neither a cone nor a cylinder is a body for which the small asymmetrical separation zones occur-flow separation begins immediately over sections of great length. For a cone with $\beta_0 > \beta_*$, the analysis carried out in Section 4 holds and indicates that the motion of a homogeneous slender "heavy" ($\rho \leqslant 0.4$) cone is unstable because $l_c = 3/4$, l_g and $= 2/3 + \chi/18$. When $\beta_0 = \beta_*$, large separation zones are formed and numerical calculations are necessary; when $\beta_0 < \beta_*$, as is generally the case for non-convex generators, an apparent paradox arises: according to the criterion of separation, the pointed nose part of the body is not wetted by the medium. This paradox is removed if it is recognized that it is a consequence of the roughness of the adopted "first-approximation" model, and, as the next step, it is necessary to introduce the dependence of the critical angle on the curvature of the generator of the body of revolution, and , in this context, to assume that this dependence is taken into account implicity (by the averaging method) and for each specific body the value of β_* is constant.

It can be shown that a cylinder is an absolutely stable shape, irrespective of the position of the centre of mass.

A body formed by revolution of part of a parabola is the simplest case in which the flow with a small separation zone is allowed. Thus, when separation occurs from the edge (see Fig. 1; the shadow zone is shown hatched)

$$\beta = \beta_0 - hg \equiv 2 - \beta_* - 2(1-\beta_*)l; \quad \beta_1 \geqslant 0, \quad h > 0 \Rightarrow 0 \leqslant \beta_* < 1; \quad R(0) = 0, \quad R(1) = 1$$

we obtain

$$l_s = l_g + \lambda\delta_1[1+\chi\delta_1], \quad l_g = 1 - \delta_1 + \chi(\delta_1^2 - 2\delta_1 + \delta_2)$$

$$\lambda = \frac{\varkappa + \beta_*^2}{\varkappa + 2 - 2\beta_* + \beta_*^2}, \quad \delta_1 = 1 - p_1 = \frac{1}{5}\left\{\frac{\beta_0^2}{6} + \frac{\beta_0}{2} + 1\right\}, \quad \delta_2 = 1 - p_2 = \frac{1}{3}\left\{\frac{\beta_0^2}{10} + \frac{2\beta_0}{5} + 1\right\}$$

$$\frac{\varkappa}{\varkappa+2} \leqslant \lambda < 1, \quad \frac{1}{3} < \delta_1 \leqslant \frac{8}{15}, \quad \frac{1}{2} < \delta_2 \leqslant \frac{11}{15}, \quad 0 \leqslant \beta_* < 1$$

Numerical example: $l_g = 0.484$ and $l_s = 0.696$ with $\beta_* = 0$, $\zeta = 2.768$ and $\varkappa = 1$, which indicates the importance of the effect of separation in determining the critical values. Furthermore, they depend on the axial velocity v_x so that in the range $0 < v_x < \infty$ with $\beta_* \ll 1$ there is a certain range of l_s values: $0 < l_* < l_s < l_{**} < 1$. When the velocity increases, the values of $\varkappa$, ψ and l_s decrease. Consequently, if the centre of mass of the body is positioned so that $l_* < l_c < l_{**}$, there is a critical velocity $v_*(l_c)$ such that, when $v_x > v_*$, l_c will be greater than $l_s(v_x)$ and the motion will be unstable, when $v_x < v_*$, the opposite assertion holds.

Preliminary analysis indicates the existence of other stable conditions of motion on passing through the critical values ($l_c > l_s, l_g$). The limiting condition $l_c = l_s$ means that the motion rapidly settles in the mode $\omega = \omega_*$ and $\eta = \eta_*$, where ω_* and η_* are constants. The trajectory of this motion is a circle of large radius. If $l_c > l_s$, the perturbations increase exponentially, and the stable trajectory will be a twisting spiral. The number of stable modes changes: the direction of rotation in the plane may be different. Consequently, the critical values l_s and l_g are bifurcation points. It is qualitatively clear to what extent these conclusions will remain valid when examining decelerated motion in the direction of the body axis. Thus, in principle it is possible to design bodies (penetrators) moving along certain prescribed curvilinear trajectories.

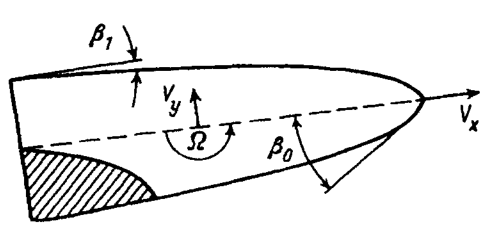

Fig. 1.

6. A PROFILE WITH SEVERAL SMALL SEPARATION ZONES

For non-convex bodies and/or bodies with special corner points one or simultaneously several asymmetrical separation zones can appear in the vicinities of a corner or a point of inflection of the generator with arbitrarily small perturbations at points $l = l_1 \leqslant l_w \leqslant 1$, $i = 2, 3, \ldots, i_0$ (edges $i = 0, 1$ can be included), where $\psi = \psi_i$, In the approximation of infinitesimal separation zones, it is possible to obtain the contribution of additional forces by the superposition method

$$A_{11} = -A_0, \quad A_{22} = A_1 - \zeta - l_c A_0, \quad A_{12} = A_1 - l_c A_0, \quad A_{22} = 2l_c A_1 - A_2 - l_c A_0^2$$

$$A_k = p_k + \sum_i l_i^k \psi_i, \quad p_k = 2\int_0^{l_w} l^k \; RdR, \quad k = 0, 1, 2$$

and to prove the inequality $B < 0$, if $\beta(l) \geqslant 0$. Assertion 1 holds, while the critical length is expressed by the formula

$$l_s = (\zeta A_0)^{-1}(A_0 A_2 - A_1 + \zeta^2 A_1)$$

The result extends to cases of angles of separation $\beta_* < 0$ and more general configurations, while the invariant $B < 0$. Analysis indicates that, the closer the separation zone is to the nose of the body, the smaller the increase in the stability margin, or it becomes negative. Thus, in the case of complete wetting, with the exception of a small asymmetrical zone in the vicinity of the tip $l = l_1 \cong 0$, we shall have

$$l_s = l_g - \frac{\psi_1(\zeta - p_1)}{\zeta p_0(\psi_1 + p_0)}$$

In typical cases ($\zeta > p_1$) the stability margin decreases compared with a scheme of continuous flow. For "light" bodies ($\zeta < p_1$) the opposite conclusion holds.

If $\beta(l) < \beta_*$, separation from the lateral surface begins only for finite perturbations lying outside a certain vicinity of the point $H = 0$. Then the stability margin, calculated by the scheme of continuous flow, is not changed even when small separation zones arise, since the additional (non-linear) terms governed by this effect on the right-hand sides of Eqs (3.1) prove to be smallest in order of magnitude.

7. A CENTRALLY SYMMETRICAL CONTOUR

Retaining the inequality $\beta' \leqslant 0$, we shall examine the motion of a body whose radius of cross-section in the direction from the tip of the body initially increases monotonically and then decreases monotonically, and the flow will be continuous if $\beta_* \leqslant \min \beta(l) = \beta_1 < 0$. In this case, the inequality $B < 0$ can break down and qualitatively different situations can arise. Thus, for a centrally symmetrical convex contour

$$R(l) = R(1 - l), \quad p_0 = 0, \quad p_1 = p_2 = -p < 0$$

when the conditions $\beta_* \leqslant \beta_1, R_1 = 0$ (a pointed body) and $\beta_* < \beta_1, R_1 > 0$ (a blunted body) (7.1) are satisfied a vicinity of the point $\eta = \omega = 0$ exists such that separation either does not arise or is insignificant. Then

$$2B = jp(1 - 2l_c), \quad D = -jp(\zeta + P) < 0$$

In the case of the start of separation from the blunted rear edge with $\beta_* = \beta_1$, we have

$$2B = jp(1 - 2l_c) - \psi(1 + l_c^2), \quad D = -jp(\zeta + P) + \psi(\zeta u_c + P)$$

$$u_c = 1 - l_c, \quad \psi = \frac{\varkappa + \beta_*^2}{2\beta'(1)} R_1$$

The following assertion follows from the results of the stability analysis (Section 3).

Assertion 2. Rectilinear motion of an elongated geometrically centrally symmetrical body, under conditions of continuous flow (7.1), is Lyaponov unstable (an unstable focus) irrespective of the position of the centre of mass over the length of the body. When separation occurs on the lateral surface, different situations are possible. Thus, in the case of separation from the rear edge ($\beta_* = \beta_1, R_1 > 0$) for small values of the parameter ψ ($0 < \psi < P$) the instability indicated above is retained; if $P < \psi < P + \zeta$, then, from the condition $D = 0$, the critical value l_s is determined and Assertion 1 holds; when $\psi > (P + \zeta)$ rectilinear motion is stable for any position of the centre of mass over the length of the body.

The strength of the medium is unimportant when analysing the stability of motion of slender bodies under conditions of continuous flow, and consequently Assertion 2 relates to the motion of bodies in an inviscid fluid. Therefore, shape asymmetry of creatures for which motion in water is an important condition for their existence is due to the need to ensure stability of rectilinear motion by inertia.

For the optimum design of bodies, a useful conclusion is that maximum stability margin as achieved by convex bodies strictly monotonically increasing radius in the direction from the tip of the body to the rear edge, where the generator should have an angle of inclination to the body axis that is similar to the average angle of separation for the trajectory of motion and a small (but non-zero) curvature.

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